Binomial Distribution

Sadia Ghulam Rasool (56)
Samreen Burki (57)
Shawal Naveed (59)
Maaz Abdul Ahad (26)
Owais Bin Kamal (54)

Binomial Distribution

What is Binomial Distribution?

- Binomial distribution is a common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters. It summarizes the number of trials when each trial has the same chance of attaining one specific outcome.

Binomial Distribution

Bernoulli Trials.

When an Experiment with only two possible outcomes is repeated in such a way that the probabilities of the two outcomes remain constant from trial to trial, the trials are called Bernoulli Trials.

Binomial Experiment

An experiment of a fixed number "n" of Bernoulli trials is called binomial experiment

or

An experiment can be called a binomial experiment only when it possesses the following properties.

a) The number of trials must be a fixed number "n"

b) Each trial must result in either a success or a failure

c) The probability p of success must remain constant from trial to trial

d) Every trial must be independent of the others

Binomial Random variable

The number X of successes in n trials of a binomial experiment is called a binomial random variable and the variable takes the values $x = 0, 1, 2, 3, \ldots\ n$ with probabilities $P(X=x)$

What makes X a binomial random variable?

- For example we flip a coin and $P(H) = 0.6$ and $P(T) = 1 – 0.6 = 0.4$. Here X is the number of heads after 10 flips.
- It is made up of finite number of Independent Trials
- Each trial has one or two discrete outcomes, it either can be classifies as a success and failure
- Random variable has fixed number of trials
- The probability of success in each trial remains constant.

Example

60% of people who purchase sports cars are men. If 10 sports car owners are randomly selected, find the probability that exactly 7 are men.

Here;

$p = 0.6$

$q = 0.4$

$X = 7$

$n = 10$

Binomial Probability Mass Function

- A rule / formula which is used to calculate the probability of x successes in "n" Bernoulli trials where p is the probability of a success in a .single trial and q is the probability of its failure such that p

$$P(X = x) = f(x) = {}^{n}C_{x}\,p^{x}.\,q^{n-x} \quad ; \quad x = 0, 1, 2, \ldots n$$

called Binomial P.M.F. in the form of formula. It may be written in tabular form as

x	0	1	2	…… …	n
P (X = x)	q^n	${}^{n}C_1 p^1 q^{n-1}$	${}^{n}C_2 p^2 q^{n-2}$	… ..	p^n

EXAMPLE QUESTION

If X is the binomial random variable in a binomial experiment with $p=\frac{3}{4}$ and $n=4$ find $P(X=2)$.

Solution:

Since $P(X=x)={}^nC_x\,p^x \cdot q^{n-x}$

$$p=\frac{3}{4} \quad \text{and} \quad q=1-p=\frac{1}{4}$$

Now by putting we get

$$P(X=2)={}^4C_2\left(\frac{3}{4}\right)^2\cdot\left(\frac{1}{4}\right)^{4-2}$$

$$P(X=2)=6\left(\frac{9}{16}\right)\left(\frac{1}{16}\right)=\frac{54}{256}$$

EXAMPLE QUESTION

The probability that a patient recovers from a delicate heart operation is 0.85. What is the probability that exacly 3 of the next 4 patients having this operation survive.

Solution :

$P(\text{ survive }) = P(\text{ recovers }) = 0.85 = p$

$P(\text{ not survive }) = q = 1 - 0.85 = 0.15$

since $n = 4$

$X = 3$

$P(X = 3) = {}^4C_3 (0.85)^3 (0.15)^1$

$= 4(0.85)^3 (0.15)^1 = 0.37$

Properties of Binomial Distribution

PROPERTIES AND IMPORTANT RELATIONS OF BINOMIAL DISTRIBUTION

(1) It is a discrete type of probability distribution.

(2) It has two parameters 'n' and 'p'.

(3) The Probabilities of the Binomial Random Variable X can be derived from the Binomial expansion: $(q + p)^n$

(4) $\sum_{x=0}^{n} {}^nC_x \, p^x q^{n-x} = (q+p)^n = 1$

and $f(x) \geq 0$ for all x = 0, 1, 2, ... n

(5) mean = np , variance = npq , S.D. = $\sqrt{npq}$

(6) mean > variance

(7) First four moments about origin are:

$$\mu_1' = np = mean$$

$$\mu_2' = n(n-1)p^2 + np$$

$$\mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$$

$$\mu_4' = n(n-1)(n-2)(n-3)p^4 + 6n(n-1)(n-2)p^3 + 7n(n-1)p^2 + np$$

(8) First four Central moments are:

$\mu_1 = 0$ always

$$\mu_2 = npq = \sigma_2$$

$$\mu_3 = npq(q-p)$$

$$\mu_4 = npq[1 + 3(n-2)pq]$$

Properties of Binomial Distribution

(9) $\beta_1 = \frac{(q-p)^2}{npq} = \frac{(1-2p)^2}{npq}$ and $\gamma_1 = \sqrt{\beta_1}$

$\beta_2 = 3 + \frac{1-6pq}{npq}$ and $\gamma_2 = \beta_2 - 3$

(10) If $p = q$ or if $p = \frac{1}{2}$ then distribution becomes symmetrical.

If $p > \frac{1}{2}$ the distribution becomes negatively skewed

If $p < \frac{1}{2}$ the distribution becomes positively skewed

EXAMPLE QUESTION

In a Binomial distribution the mean = 24 and S.D. = 4. Find n and p.

Solution:

$np = 24$

$S.D. = \sqrt{npq} = 4$

$npq = 16$

$24q = 16$

$q = \frac{16}{24} = \frac{2}{3}$, $p = \frac{1}{3}$

Therefore $n = 72$

THANK YOU